

Polarised deep inelastic scattering accompanied by a forward jet as a probe of the $\ln^2(1/x)$ resummation

J. KWIECIŃSKI,¹ B. ZIAJA²

*Department of Theoretical Physics,
H. Niewodniczański Institute of Nuclear Physics, Cracow, Poland*

Abstract

We argue that the production of forward jets in polarised deep inelastic scattering can be a useful tool for probing the double $\ln^2(1/x)$ resummation effects which control the polarised deep inelastic scattering for small values of the Bjorken parameter x . We solve the corresponding integral equations generating the double $\ln^2(1/x)$ resummation and calculate the differential structure function describing the forward jet production in the small x regime which can possibly be probed by the polarised HERA measurements. We show that these structure functions should exhibit the characteristic increase with decreasing x/x_J , where x_J denotes the longitudinal momentum fraction of the parent proton carried by a jet, and we quantify this increase.

The idea of studying the deep inelastic ep scattering with an identified forward jet as a probe of the low x behaviour of QCD was proposed by Mueller [1]. Since then it has been successfully applied to the case of unpolarised deep inelastic scattering (DIS) [2, 3]. The Mueller's proposal was to study the unpolarised deep inelastic events (x, Q^2) containing an identified forward jet (x_J, k_J^2) , where the longitudinal momentum fraction x_J of the proton carried by a forward jet and jet transverse momentum squared k_J^2 were assumed to fulfill the conditions :

$$x_J \gg x, \tag{1}$$

$$k_J^2 \sim Q^2. \tag{2}$$

¹e-mail: jkwiecin@solaris.ifj.edu.pl

²e-mail: beataz@solaris.ifj.edu.pl

Here, as usual, $Q^2 = -q^2$, where q is the four momentum transfer between electrons in the deep inelastic ep scattering, and x is the Bjorken parameter, i.e. $x = Q^2/(2pq)$ with p denoting the four momentum of the proton.

The first assumption $x_J \gg x$ implies that one probes the QCD dynamics in the low x/x_J region (the quantities which are measured depend on the ratio x/x_J). The second assumption $k_J^2 \sim Q^2$ guarantees suppression of the standard leading order (LO) DGLAP evolution [1, 2, 3] and restricted penetration of the non-perturbative region by the small x resummation [4]. Let us recall that the small x behaviour of (unpolarised) deep inelastic scattering should be controlled by the QCD pomeron which in perturbative QCD is described by the Balitskij, Fadin, Kuraev, Lipatov (BFKL) equation [5]. This equation generates the leading $\ln(1/x)$ resummation, and it corresponds to the sum of ladder diagrams with the (reggeised) gluons along the chain. Unlike the LO DGLAP equations which correspond to ladder diagrams with ordered transverse momenta, the transverse momenta of the gluons along the BFKL ladder are not ordered. In fact, the small x behaviour generated by the BFKL equation is linked with the diffusion of the transverse momenta both towards the infrared and the ultraviolet regions. It is this diffusion which is probed in the kinematical configurations which have comparable scales on both sides of the ladder, i.e. $k_J^2 \sim Q^2$ for the forward jet production.

In this paper we would like to apply the idea of forward jet measurement to the case of polarised deep inelastic ep scattering in the region of small values of x which can be probed at the possible polarised HERA measurements [6]. The pomeron contribution decouples from the polarised deep inelastic scattering, and its small x limit is sensitive to the novel effects of the double $\ln^2(1/x)$ resummation [7, 8].

The relevant dynamical quantity in the case of jet production in polarised deep inelastic scattering is the differential spin structure function $x_J \frac{\partial g_1}{\partial x_J \partial k_J^2}$. It is linked to the corresponding differential cross-section in the standard way:

$$\frac{\partial^4 \sigma}{\partial x \partial Q^2 \partial x_J \partial k_J^2} = \frac{8\alpha_e^2 \pi}{Q^4} \frac{\partial^2 g_1}{\partial x_J \partial k_J^2} y(2-y), \quad (3)$$

where α_e denotes the electromagnetic coupling constant, and y describes the energy fraction of incoming electron carried by the interacting virtual photon. In equation (3) we have omitted terms proportional to $\gamma^2 = 4M^2 x^2 / Q^2$, where M denotes nucleon mass, which are negligible at small x . The cross section in formula (3) corresponds, as usual, to the difference between the cross-sections for antiparallel and parallel spin orientations [9]. Similarly as for the unpolarised case, restrictions (1), (2) applied to the polarised DIS forward jet events allow one to neglect the effects of DGLAP evolu-

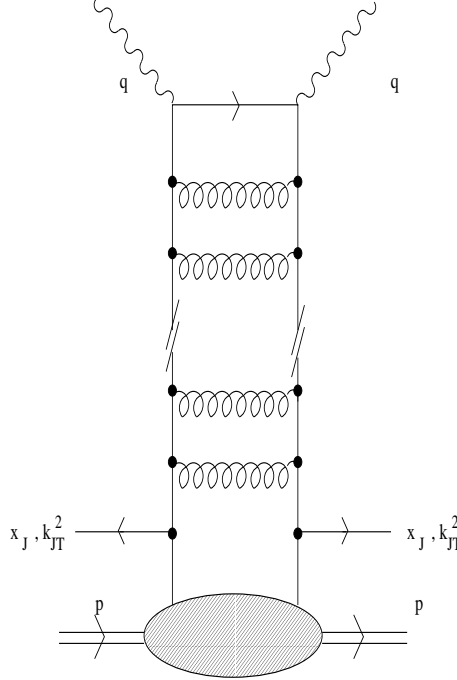


Figure 1: An example of the ladder diagram contributing to the differential spin structure function.

tion and to concentrate on the low x/x_J behaviour of $x_J \frac{\partial^2 g_1}{\partial x_J \partial k_J^2}$. Low x/x_J behaviour of differential spin structure function is dominated by double logarithmic $\ln^2(1/x)$ (or rather $\ln^2(x_J/x)$) contributions i.e. by those terms of the perturbative expansion which correspond to the powers of $\ln^2(x_J/x)$ at each order of the expansion, similarly as for the DIS spin dependent structure function g_1 [7, 8]. In the following we will apply an approach to the double $\ln^2(1/x)$ resummation based on unintegrated parton distributions [10, 11, 12, 13].

The full contribution to the double $\ln^2(1/x)$ resummation comes from the ladder diagrams with quark and gluon exchanges along the ladder (see e. g. Fig. 1) and the non-ladder bremsstrahlung diagrams [14]. The latter ones are obtained from the ladder diagrams by adding to them soft bremsstrahlung gluons or soft quarks [7, 8, 14], and they generate the infrared corrections to the ladder contribution. However, it was shown in ref. [13] that the effects of non-ladder contributions considered for DIS spin dependent structure function g_1 are visible only for $x < 10^{-3}$. In what follows we assume $x/x_J > 10^{-3}$ and limit ourselves to the contributions from ladder diagrams which dominate in this region.

The relevant region of phase space generating the double $\ln^2(1/x)$ resummation from ladder diagrams corresponds to ordered k_n^2/x_n , where k_n^2 and x_n denote respec-

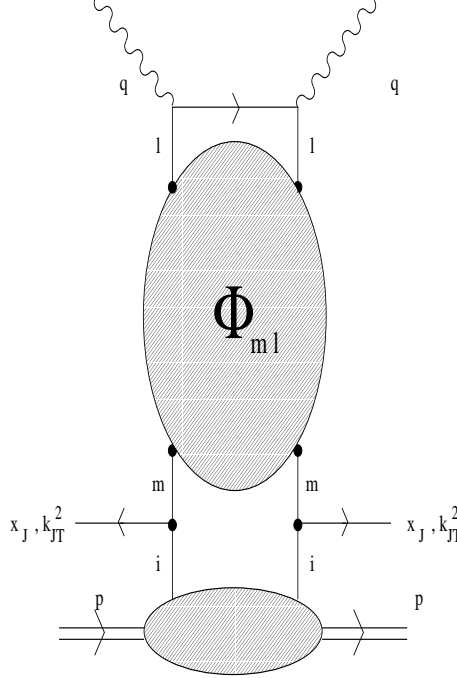


Figure 2: Diagrammatic representation of formula (4) for the differential structure function $x_J \frac{\partial^2 g_1}{\partial x_J \partial k_J^2}$ describing the (forward) jet production in polarised deep inelastic scattering.

tively the transverse momenta squared and longitudinal momentum fractions of the proton carried by partons exchanged along the ladder [15]. In consequence, the effects of the double $\ln^2(1/x)$ resummation should be still visible in the kinematical configuration having comparable scales on both sides of the ladder (i.e. $k_J^2 \sim Q^2$ for jet production) in contrast to the LO DGLAP evolution alone which corresponds to ordered transverse momenta.

The formula for the differential structure function can be written in the following form (see Fig. 2) :

$$x_J \frac{\partial^2 g_1}{\partial x_J \partial k_J^2} = \bar{\alpha}_s(k_J^2) \sum_{iml} \Delta p_i(x_J, k_J^2) \Delta P_{mi}(0) \Phi_{ml}\left(\frac{x}{x_J}, k_J^2, Q^2\right), \quad (4)$$

where $\Delta p_i(x_J, k_J^2)$ are the (integrated) spin dependent parton distributions in the proton. The quantities $\Delta P_{im}(0) \equiv \Delta P_{im}(z = 0)$, where $\Delta P_{im}(z)$ (for quarks $\Delta P_{im}(z) = \delta_{im} \Delta P_{qq}$) denote the LO splitting functions describing evolution of spin dependent parton densities. The indices i and m numerate quarks, antiquarks and gluons and the index l quarks and antiquarks respectively. Finally, $\bar{\alpha}_s = \alpha_s/2\pi$. Equation (4) was derived, assuming strong ordering of transverse momenta ($k_i^2 \ll k_J^2 \sim k_m^2$) and of the longitudinal momentum fractions ($x_m \ll x_i \sim x_J$) at the jet production vertex (see

Fig. 2).

The functions $\Phi_{ml}(\frac{x}{x_J}, k_J^2, Q^2)$ are related to the unintegrated quark and antiquark distributions $f_{ml}(\frac{\bar{x}}{x'}, k_J^2, k_f^2)$ in the parton m , where k_f^2 denotes the transverse momentum squared of the quark (antiquark):

$$\Phi_{ml}(\frac{x}{x_J}, k_J^2, Q^2) = \frac{1}{2} e_l^2 \int^{\bar{W}^2} \frac{dk_f^2}{k_f^2} \int_{\bar{x}}^{x_J} \frac{dx'}{x'} f_{ml}(\frac{\bar{x}}{x'}, k_J^2, k_f^2), \quad (5)$$

where

$$\bar{x} = x \left(1 + \frac{k_f^2}{Q^2} \right), \quad (6)$$

$$\bar{W}^2 = Q^2 \left(\frac{x_J}{x} - 1 \right). \quad (7)$$

We show later that the integration limits in eq. (5) are further restricted by the ordering of k_n^2/x_n . In particular this ordering will provide the infrared cut-off for k_f^2 .

It is convenient to introduce the non-singlet and singlet combination of parton distributions

$$f_m^{NS} = \sum_{l=1}^{N_f} \left(\frac{e_l^2}{\langle e^2 \rangle} - 1 \right) f_{ml}, \quad (8)$$

$$f_m^S = \sum_{l=1}^{N_f} f_{ml}, \quad (9)$$

where $\langle e^2 \rangle = \frac{1}{N_f} \sum_{l=1}^{N_f} e_l^2$, and N_f denotes the number of active flavours ($N_f = 3$).

Restricting ourselves to ladder diagrams in the double $\ln^2(1/\xi)$ approximation, we get the following equations for the functions $f_m^{NS}(\xi, k_J^2, k_f^2)$ and $f_m^S(\xi, k_J^2, k_f^2)$:

$$f_m^{NS}(\xi, k_J^2, k_f^2) = f_m^{NS0}(\xi, k_J^2, k_f^2) + \bar{\alpha}_s(\mu^2) \Delta P_{qq}(0) \int_{\xi}^1 \frac{d\xi'}{\xi'} \int_0^{k_f^2/z} \frac{dk^2}{k^2} f_m^{NS}(\xi', k_J^2, k^2) \Theta(k^2 - k_J^2 \xi'), \quad (10)$$

$$f_m^S(\xi, k_J^2, k_f^2) = f_m^{S0}(\xi, k_J^2, k_f^2) + \bar{\alpha}_s(\mu^2) \int_{\xi}^1 \frac{d\xi'}{\xi'} \int_0^{k_f^2/z} \frac{dk^2}{k^2} (\Delta P_{qq}(0) f_m^S(\xi', k_J^2, k^2) + \Delta P_{qg}(0) f_m^g(\xi', k_J^2, k^2)) \Theta(k^2 - k_J^2 \xi'), \quad (11)$$

$$f_m^g(\xi, k_J^2, k_f^2) = f_m^{g0}(\xi, k_J^2, k_f^2) + \bar{\alpha}_s(\mu^2) \int_{\xi}^1 \frac{d\xi'}{\xi'} \int_0^{k_f^2/z} \frac{dk^2}{k^2} (\Delta P_{gq}(0) f_m^S(\xi', k_J^2, k^2) + \Delta P_{gg}(0) f_m^g(\xi', k_J^2, k^2)) \Theta(k^2 - k_J^2 \xi'), \quad (12)$$

where $z = \xi/\xi'$. The inhomogeneous terms are given by the following formulae:

$$f_m^{NS0}(\xi, k_J^2, k_f^2) = \delta(k_f^2 - k_J^2)\delta(\xi - 1) \left(\frac{e_m^2}{\langle e^2 \rangle} - 1 \right) (\delta_{mq_m} + \delta_{m\bar{q}_m}), \quad (13)$$

$$f_m^{S0}(\xi, k_J^2, k_f^2) = \delta(k_f^2 - k_J^2)\delta(\xi - 1)(\delta_{mq_m} + \delta_{m\bar{q}_m}), \quad (14)$$

$$f_m^{g0}(\xi, k_J^2, k_f^2) = \delta(k_f^2 - k_J^2)\delta(\xi - 1)\delta_{mg}. \quad (15)$$

We have solved equations (10), (11) and (12) for two choices of the scale μ^2 which allow analytical solution of these equations: $\mu^2 = (k_J^2 + Q^2)/2$ and $\mu^2 = k_f^2/\xi$. In the first case the coupling $\alpha_s(\mu^2)$ does not change along the ladder, and it is a fixed parameter. In what follows we shall call this choice "the fixed coupling case". Let us remind that this choice of the scale follows the convention usually adopted in the case of forward jet production in the unpolarised deep inelastic scattering [2]. In the second case $\mu^2 = k_f^2/\xi$ the coupling changes along the ladder, and so we shall call this choice the "running coupling case". One expects that the scale $\mu^2 = k_f^2/\xi$ is presumably too large, and that this choice will lead to an underestimate of the effect. The more natural choice $\mu^2 = k_f^2$ does not, however, allow analytical solution. One can expect that the solution of equations (10) - (12) with $\mu^2 = k_f^2$ should lie between the two solutions corresponding to $\mu^2 = (k_J^2 + Q^2)/2$ and $\mu^2 = k_f^2/\xi$.

For the case of fixed $\bar{\alpha}_s$ the solutions of equations (10), (11) and (12) for the functions $f_m^{NS}(\xi, k_J^2, k_f^2)$ and $f_m^S(\xi, k_J^2, k_f^2)$ take the following form :

$$f_m^{NS}(\xi, k_J^2, k_f^2) = f_m^{NS0}(\xi, k_J^2, k_f^2) + \frac{\bar{\alpha}_s \lambda}{k_J^2} (\delta_{mq_m} + \delta_{m\bar{q}_m}) \left(\frac{e_m^2}{\langle e^2 \rangle} - 1 \right) I_0[2\sqrt{\bar{\alpha}_s \lambda} y(t+y)] \Theta(k^2 - k_J^2 \xi), \quad (16)$$

$$\begin{aligned} f_m^S(\xi, k_J^2, k_f^2) &= f_m^{S0}(\xi, k_J^2, k_f^2) \\ &+ \frac{\bar{\alpha}_s \lambda_0^+}{k_J^2} \frac{c_m^-}{\lambda_0^- - \lambda_0^+} \Delta_{qq}(0) I_0[2\sqrt{\bar{\alpha}_s \lambda_0^+} y(t+y)] \Theta(k_f^2 - k_J^2 \xi) \\ &+ \frac{\bar{\alpha}_s \lambda_0^-}{k_J^2} \frac{c_m^+}{\lambda_0^- - \lambda_0^+} \Delta_{qq}(0) I_0[2\sqrt{\bar{\alpha}_s \lambda_0^-} y(t+y)] \Theta(k_f^2 - k_J^2 \xi), \end{aligned} \quad (17)$$

where $I_0(z)$ denotes the modified Bessel function,

$$t = \ln \left(\frac{k_f^2}{k_J^2} \right), \quad (18)$$

$$y = \ln \left(\frac{1}{\xi} \right), \quad (19)$$

$$\lambda = \Delta P_{qq}(0), \quad (20)$$

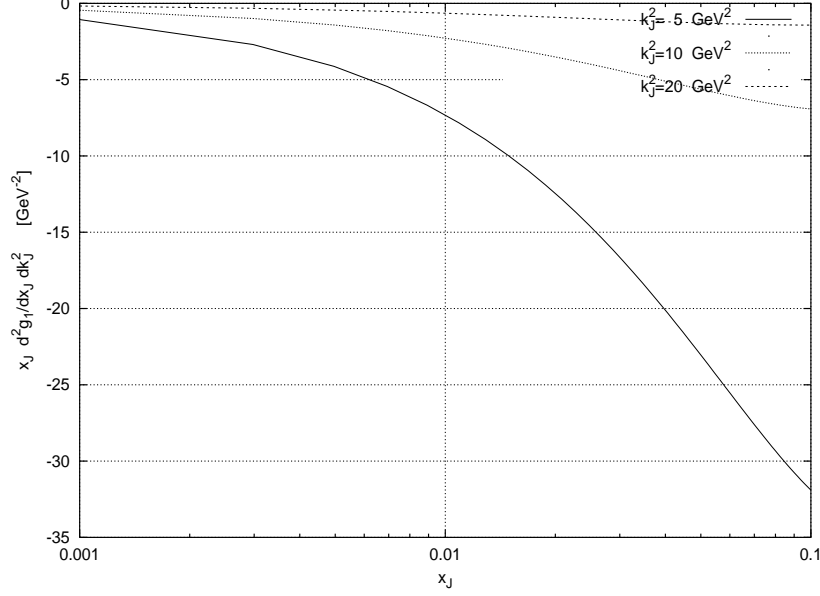


Figure 3: The differential spin structure function $x_J \frac{\partial g_1}{\partial x_J \partial k_J^2}$ for the fixed $\bar{\alpha}_s$ case plotted as the function of the longitudinal momentum fraction x_J carried by a jet. We show our predictions for three values of the transverse momentum squared k_J^2 of the jet i.e. for 5 GeV², 10 GeV² and 20 GeV². The calculations were performed for $Q^2 = 10 \text{ GeV}^2$ and $x = 10^{-4}$.

λ_0^+, λ_0^- are the eigenvalues of matrix $\mathbf{\Delta P(0)} \equiv \begin{pmatrix} \Delta P_{qq}(0) & \Delta P_{qg}(0) \\ \Delta P_{gq}(0) & \Delta P_{gg}(0) \end{pmatrix}$:

$$\lambda_0^\pm = \frac{\Delta P_{qq}(0) + \Delta P_{gg}(0) \pm \sqrt{(\Delta P_{qq}(0) - \Delta P_{gg}(0))^2 + 4\Delta P_{qg}(0)\Delta P_{gq}(0)}}{2}, \quad (21)$$

and coefficients c_m^+, c_m^- are defined as :

$$\begin{aligned} c_m^+ &= \delta_{mg} - (\delta_{mqm} + \delta_{m\bar{q}m}) \frac{\lambda_0^+ - \Delta P_{qq}(0)}{\Delta P_{qg}(0)}, \\ c_m^- &= -\delta_{mg} + (\delta_{mqm} + \delta_{m\bar{q}m}) \frac{\lambda_0^- - \Delta P_{qq}(0)}{\Delta P_{qg}(0)}. \end{aligned}$$

This form is typical for the double logarithmic asymptotics (cf. [16] and references therein). It has also been obtained for fragmentation functions [17].

The solution for the case of running $\bar{\alpha}_s = \bar{\alpha}_s(k_f^2/\xi)$ for the non-singlet and the singlet components read :

$$\begin{aligned} f_m^{NS}(\xi, k_J^2, k_f^2) &= f_m^{NS0}(\xi, k_J^2, k_f^2) \\ &+ \frac{\bar{\alpha}_s(k_f^2/\xi)\lambda}{k_J^2} (\delta_{mqm} + \delta_{m\bar{q}m}) \left(\frac{e_m^2}{\langle e^2 \rangle} - 1 \right) I_0 [2\sqrt{c\lambda y \ln \frac{\rho + \rho_0}{\rho_0}}] \Theta(k^2 - k_J^2 \xi), \quad (22) \end{aligned}$$

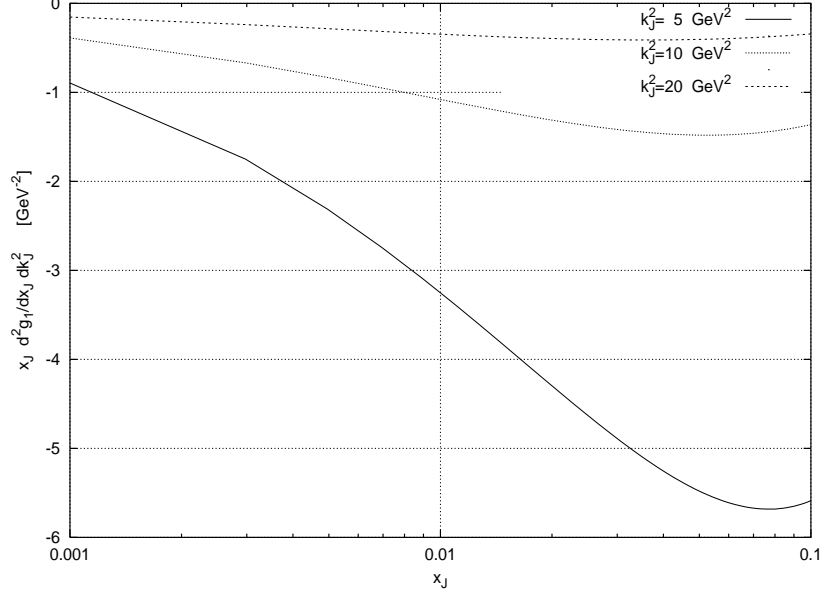


Figure 4: The differential spin structure function $x_J \frac{\partial g_1}{\partial x_J \partial k_J^2}$ for the running $\bar{\alpha}_s$ case plotted as the function of the longitudinal momentum fraction x_J carried by a jet. We show our predictions for three values of the transverse momentum squared k_J^2 of the jet i.e. for 5 GeV^2 , 10 GeV^2 and 20 GeV^2 . The calculations were performed for $Q^2 = 10 \text{GeV}^2$ and $x = 10^{-4}$.

$$\begin{aligned}
f_m^S(\xi, k_J^2, k_f^2) &= f_m^{S0}(\xi, k_J^2, k_f^2) \\
&+ \frac{\bar{\alpha}_s(k_f^2/\xi) \lambda_0^+}{k_J^2} \frac{c_m^-}{\lambda_0^- - \lambda_0^+} \Delta_{qg}(0) I_0[2\sqrt{c\lambda_0^+ y \ln \frac{\rho + \rho_0}{\rho_0}}] \Theta(k^2 - k_J^2 \xi) \\
&+ \frac{\bar{\alpha}_s(k_f^2/\xi) \lambda_0^-}{k_J^2} \frac{c_m^+}{\lambda_0^- - \lambda_0^+} \Delta_{qg}(0) I_0[2\sqrt{c\lambda_0^- y \ln \frac{\rho + \rho_0}{\rho_0}}] \Theta(k^2 - k_J^2 \xi), \quad (23)
\end{aligned}$$

where :

$$\rho = \ln \frac{k_f^2}{k_J^2 \xi}, \quad (24)$$

$$\rho_0 = \ln \frac{k_J^2}{\Lambda_{QCD}^2}, \quad (25)$$

and

$$c = \frac{2}{11 - 2/3 N_f}. \quad (26)$$

Having calculated the unintegrated parton distributions for both the fixed and the running $\bar{\alpha}_s$ case, one may express the double logarithmic contribution $\sum_l \Phi_{ml}$ in (4) as :

$$\sum_l \Phi_{ml}(\frac{x}{x_J}, k_J^2, Q^2) = \frac{1}{2} \langle e^2 \rangle \int_{x_{down}}^{x_{up}} \frac{dx'}{x'} \int_{k_{down}^2}^{k_{up}^2} \frac{dk_f^2}{k_f^2} (f_m^{NS}(\frac{\bar{x}}{x'}, k_J^2, k_f^2) + f_m^S(\frac{\bar{x}}{x'}, k_J^2, k_f^2)), \quad (27)$$

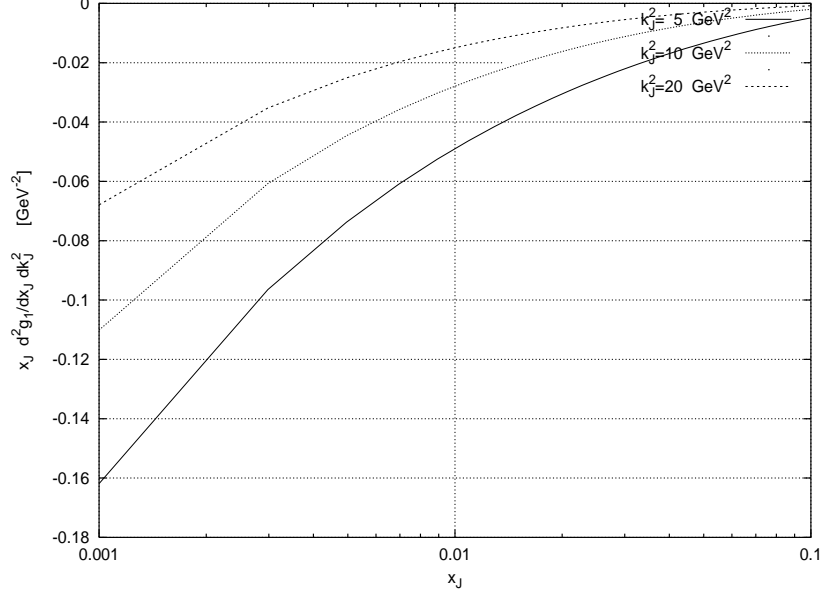


Figure 5: Born approximation for the differential spin structure function $x_J \frac{\partial g_1}{\partial x_J \partial k_J^2}$ plotted as the function of the longitudinal momentum fraction x_J carried by a jet. We show our predictions for three values of the transverse momentum squared k_J^2 of the jet i.e. for 5 GeV², 10 GeV² and 20 GeV². The calculations were performed for $Q^2 = 10 \text{ GeV}^2$ and $x = 10^{-4}$.

where the integration limits x_{up} , x_{down} , k_{up}^2 and k_{down}^2 which read :

$$\begin{aligned} x_{up} &= x_J, & x_{down} &= x \left(1 + \frac{k_J^2}{Q^2}\right), \\ k_{up}^2 &= Q^2 \left(\frac{x'}{x} - 1\right), & k_{down}^2 &= k_J^2 / \left(\frac{x'}{x} - \frac{k_J^2}{Q^2}\right) \end{aligned} \quad (28)$$

take into account restrictions implied by the integration limits in (5) and by the Θ functions in equations (16,17) etc. i. e. :

$$k_f^2 < \bar{W}^2, \quad (29)$$

$$\bar{x} < x' < x_j, \quad (30)$$

$$k_J^2 \frac{\bar{x}}{x'} < k_f^2. \quad (31)$$

The latter restriction is the most important one: it follows from the ordering of the ratio k_n^2/x_n along the ladder which, in turn, determines the infrared limit in the integration over dk_f^2 .

We have performed numerical analysis of the low x/x_J behaviour of differential structure function $\frac{\partial^2 g_1}{\partial x_J \partial k_J^2}$. It was calculated by convoluting the double logarithmic

contribution $\sum_l \Phi_{ml}$ for the fixed and running $\bar{\alpha}_s$, respectively, with the spin dependent parton distributions $\Delta p_i(x_J, k_J^2)$. The parton distributions $\Delta p_i(x_J, k_J^2)$ were obtained from the input distributions at the initial scale equal 1 GeV² parametrised as in ref. [12, 13] which were evolved using the leading order Altarelli-Parisi evolution for the spin dependent parton densities [18]. Our results are presented in Figs. 3 and 4 for the fixed and running $\bar{\alpha}_s$ case respectively. In both cases the solutions depend strongly on the value of the jet transverse momentum k_J^2 (cf. equations (16),(17),(22),(23)). One may notice that the absolute value of the differential structure function exhibits very strong increase with the increasing value of x_J (i.e. decreasing x/x_J). This increase is a direct consequence of the double $\ln^2(1/x)$ resummation summarised in equations (16),(17),(22),(23). The absolute magnitude of the differential structure function which corresponds to the running coupling case (i.e. to the scale $\mu^2 = k_J^2/\xi$ in equations (10),(11) and (12)) is significantly smaller than that one which corresponds to the fixed coupling, i.e. $\mu^2 = (k_J^2 + Q^2)/2$. The former scale is presumably too large, and the latter case may be regarded as the more realistic one.

We have also calculated the differential structure function in Born approximation which corresponds to the replacement of functions $f_m^{NS}(\xi, k_J^2, k^2)$ and $f_m^S(\xi, k_J^2, k^2)$ by inhomogeneous terms $f_m^{NS0}(\xi, k_J^2, k^2)$ and $f_m^{S0}(\xi, k_J^2, k^2)$ defined by equations (13) and (14). Results of this calculations are summarised in Fig. 5. We may see that the absolute magnitude of the differential structure function with the effects of the double $\ln^2(1/x)$ resummation (see Figs. 3 and 4) taken into account is significantly larger than the absolute magnitude of the differential structure function calculated in Born approximation. Let us also note that the x_J dependence is completely different in those two cases i.e. the effects of the double $\ln^2(1/x)$ resummation generate increase of the absolute magnitude of the structure functions with increasing x_J (for fixed x), while the absolute magnitude of the structure function calculated in Born approximation decreases with increasing x_J , just following the x_J dependence of the spin dependent parton distributions $\Delta p_i(x_j, k_J^2)$. Comparison with Born approximation leads to the conclusion that the double logarithmic resummation effects should be clearly visible for $x_J > 10^{-2}$ ($x = 10^{-4}$).

To sum up, we have estimated possible effects of the double $\ln^2(1/x)$ resummation in the forward jet production in the polarised deep inelastic scattering at the small x regime which will possibly be probed at the polarised HERA. We have shown that the (absolute) magnitude of the differential spin dependent structure function strongly increases with decreasing x/x_J , and that it is significantly larger than the "background" which corresponds to Born term with the double $\ln^2(1/x)$ resummation neglected. Estimate of the expected rate of the forward jet production in polarised deep inelastic

ep scattering taking into account the HERA acceptance is in progress.

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